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Decomposition of Continuity via Locally b-Closed, b-Pre-Open & sb-Generalized Closed Sets in Topological Spaces

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ABSTRACT:
The present paper introduces the notions of locally b-closed continuous, D(c,d) continuous, b-pre continuous, b-semi continuous, contra b-continuous, sb-cotinuous & contra sb-continuous functions, The basic properties of such mappings have been found a place here. Theorems based upon the interrelations of such functions have also been discovered in this paper. It also highlights b-B-continuous as well as b-t-continuous with completely continuous functions.

Keywords: b-open, b-pre-open, b-semi-open, locally b-closed, sb-generalized closed set, t-set, b-t-set, B-set, b-B-set & D(c,d) set

1. INTRODUCTION & PREREQUISITES:

The weak forms of open sets in a topological space as semi-pre-open and b-open sets were introduced by D. Andrijevic through the mathematical papers [1,2]. The concept of generalized open sets with the introduction of semi-open sets was studied by Levine [3] and N jasted [4] investigated α-open sets and Mashhour et. al. [5] introduced pre-open sets At the same time Bourbaki [6] invented the concept of locally closed sets. Obviously, one of the most significant concepts in topology is the notion of b-open sets which was discussed by E. Ekici & M. Caldas [7] under the name γ-open set. J. Tong [9]. Introduced the concept of t-set and B-set in a topological space.

The concepts of locally b-closed, b-pre-open, b-semi-open, D(c,b) and sb-generalized closed sets play an important role in topological spaces which were defined using the closure operator (cl), b-closure operator (bcl), interior operator(int) and b-interior operator (bint) in a systemetic manner.

Definition (1.1) :

A subset A of a space (X,T) is called

(a) a semi-open [3] set if $A \subset cl(int(A))$ and semi-closed set if $int(cl(A)) \subset A$

(b) a pre-open [5] set if $A \subset int(cl(A))$ and pre-closed set if $cl(int(A)) \subset A$
Also, a regular open if $A=int(cl(A))$.

(c) an α-open [4] set if $A \subset int(cl(int(A))$ and $\alpha-$ closed set if $cl(int(cl(A))) \subset A$

The semi-pre-open set, called by D. Andrijevic was introduced under the name β-open set by M.E Abd. El-Monsef et.al. as:

Definition (1.2):
A subset A of a space (X,T) is called semi-pre-open [1] or β-open [7] set if $A\subset cl(int(cl(A)))$ and a semi-pre-closed or β-closed set if $int(cl(int(A))) \subset A$.
Now, the new class of generalized open sets given by D. Andrijevic under the name of b-open sets is stated as:

Definition (1.3) :
A subset A of a space (X,T) is called a b-open set [2] if $A \subset cl(int(A)) \cup int(cl(A))$

and a b-closed set [8] if cl (int(A)) $\cap$int(cl(A)) $\subset$ A.

All the definitions (1.1), (1.2) & (1.3) are different and independent [8].

The classes of pre-open, semi-open, α-open, semi-pre-open and b-open subsets of a space (X,T) are usually denoted by PO(X,T), SO(X,T), T^{α} , SPO(X,T) and BO(X,T) respectively. All of them are larger than T and closed under forming arbitrary unions.

In 1996, D. Andrijevic made the fundamental observation:

Prop : (1.4) : For every space (X,T)

PO(X, T) $\cup$SO(X, T) $\subseteq$ BO(X,T) $\subseteq$ SPO(X,T) holds but none of these implications can be reversed [8].

Prop. (1.5) : Characterization [8] :

(a) S is semi-pre-open iff S $\subseteq$ Sint(Scl S)

(b) S is semi-open iff S $\subseteq$ Scl(Sint S)

(c) S is pre-open iff S $\subseteq$ pint(pcl S)

(d) S is b-open iff S $\subseteq$ pcl(pint S), where S $\subseteq$ X and (X,T) is a space.

Next, J. Tong [9] introduced the concept of t-set and B-set in a topological space as :

Definition (1.6) :

A subset A of a space (X,T) is called

(a) a t-set [9] if int(A) = int(cl(A)).

(b) a B-set [9] if A = U $\cap$ V where U $\in$ T & V is a t-set.

(c) Locally closed [6] if A = U $\cap$ V where U $\in$ T & V is a closed set.

(d) Locally b-closed [10] if A = U $\cap$ V where U $\in$ T & V is a b-closed set.

(e) a D(c,b)-set if int(A) = bint(A)

(f) a b-semi-open set if A $\subset$ cl(bint(A)) and a b-semi-closed set if int(bcl(A)) $\subset$A.

(g) a b-pre-open set if A $\subset$ int(bcl(A)) and a b-pre-closed set if cl(bint(A)) $\subset$A.

(h) a sb-generalized closed set if sbcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is a b-pre-open set.

Here, sbcl(A) is the intersection of all b-semi-closed sets containingA.

(i) a b-t-set if int(A) = int(bcl(A)).

(j) a b-B-set if A = U $\cap$V where U $\in$ T and V is a b-t-set.

$\ast$ All the above concepts are independent [10]. some basic properties of such sets are enunciated in the following theorems which are the main parts of the mathematical paper [10] produced by the authors :

Theorem (1.7) : If A is a subset of an extremely disconnected space (X,T) then the following statement are equivalent :

(a) A is open

(b) A is b-open & locally closed.

Theorem (1.8) : If A $\subseteq$X where (X,T) is a space, then

[A is open] $\Leftrightarrow$ [A is a b-open set and a D(c,b) set]

Theorem (1.9) : If A be a locally b-closed subset of (X,T) then

(a) bcl(A) - A is a b-closed set

(b) [A $\cup$ (X-bcl(A))] is b-open

(c) A $\subseteq$ bint [A $\cup${ X-bcl(A)}]

Corollary: The intersection of a locally b-closed set & a locally closed set is locally b-closed.

Theorem (1.10): If (X,T) is closed under finite union of b-closed sets and A & B are sepa rated locally b-closed sets, then $A \cup B$ is locally b-closed.

Lemma (1.11): If A is an open set of (X,T), then bcl(A) = Scl(A) and int (bcl(A)) = int(cl(A)).

Corollary: For any open subset A of (X,T),

$[A \subseteq \text{int}(\text{bcl}(A))] \Leftrightarrow [A \subseteq \text{int}(\text{cl}(A))]$

i.e. A is b-preopen $\Leftrightarrow$ A is pre-open

Theorem (1.12): For a subset A of a space (X,T), the following statements are equivalent

(a) A is regular open
(b) A = int(bcl(A))
(c) A is b-pre open and b-t set

Theorem (1.13): If $A \subseteq X$ where (X,T) is a space, then

[A is regular open] $\Leftrightarrow$ [A is b-proper & sb-generalized closed]

2. DECOMPOSITION OF CONTINUITY:

This section introduces the following definitions regarding decomposition of continuity due to the different types of open as well as closed sets mentioned in the section 1 :

Definition (2.1): A mapping $f : (X,T) \to (Y, \sigma)$ from a topological space (X,T) to another space (Y, σ) is called :

(a) b-continuous if $f^{-1}(v)$ is b-open in X for each open set V of Y.
(b) Semi-continuous if $f^{-1}(V)$ is semi-open in X for each open set V of Y.
(c) Completely continuous if $f^{-1}(V)$ is regular open in X for each open set V of Y.
(d) Contra b-continuous if $f^{-1}(V)$ is b-closed in X for each open set V of Y
(e) B-continuous if $f^{-1}(V)$ is B-set in X for each open set V of Y.
(f) Locally closed continuous if $f^{-1}(V)$ is locally closed in X for each open set V of Y.
(g) Locally b-closed continuous if $f^{-1}(V)$ is locally b-closed in X for each open set V of Y
(h) D(c,d)-continuous if $f^{-1}(V)$ is D(c,b) set in X for each open set V of Y.
(i) Contra gb-continuous if $f^{-1}(V)$ is gb-closed in X for each open set V of Y.
(j) b-pre continuous $f^{-1}(V)$ is b-pre open in X for each open set V of Y.
(k) b-semi-continuous if $f^{-1}(V)$ is b-semi-open in X for each open set V of Y.
(l) b-t continuous if $f^{-1}(V)$ is b-t set in X for each open set V of Y.
(m) b-B continuous if $f^{-1}(V)$ is b-B set in X for each open set V of Y.
(n) Contra sb-continuous if $f^{-1}(V)$ is sb-generalized closed in X for each open set V of Y.

(o) sb-continuous if $f^{-1}(V)$ is sb-generalized open in X for each open set V of Y.

Example (2.2) : Let X={a,b,c}, T={ϕ,{a}, {a,c}, X}, then the simple computation gives that BO(X,T)={ ϕ ,{a},{a,b},{a,c},X}

The class of all D(c,b) sets = { ϕ,{a},{b},{c}, {a,c}, {b,c}, X}

The class of Locally closed sets = { ϕ,{a}, {b}, {c}, {a,c} , {b,c}, X}

Let Y = {l,m,n,p}, σ={ ϕ, {l} , {m}, {l,m}, {m,n,p}, Y}

(i) Let f : (X,T) → (Y, σ) be given by f(a) = l, f(c) = m, f(b)= n of course f is not b- continuous but D(c,b) - continuous as well as locally closed continuous.

(ii) Let g: (X,T) → (z, η) be given by g(b) = l, g(c)=m, g(a)=n

Where Z= {l,m,n} & η = { ϕ, {l} ,{m},{l,m}, Z};

Then g is contra b-continuous & locally closed continuous

Again, Locally b-closed sets of (X,T) are P(X). So, g is also locally b-closed continuous.

Example (2.3) : Let X = {a,b,c,d}, T={ϕ,x,{c},{d},{c,d},{a,c,d},{b,c,d}}

Then BO(X,T) = P(X) - { {a}, {a,b}} = SBGO(X,T) & BPO(X,T) = T

The collection of all sb-generalized open sets = P(X) - {{a},{b},{a,b}}

The collection of all sb-generalized closed sets =P(X) - {{c,d},{a,c,d},{b,c,d}

Let Y={l,m,n,p}, σ ={ϕ,Y,{m},{l,m},{m,n,p}}

(i) Let f : (X,T) → (Y, σ) be given by f(a) = m, f(b) = l = f(d) and f(c) = n

Of course f is contra sb-continuous but not sb-continuous. Also f is locally closed continuous & locally b-closed continuous. At the same time f is not b-pre-continuous

(ii) Let g : (X,T) → (Y, σ) be given by g(a)=m,g(b)=l, g(c)=n, g(d)=p.

Then g is neither sb-continuous nor contra sb-continuous.

(iii) Let g^*: (X, T) → (Y, σ) given by g^* (a)=p, g^* (b)=l, g^* (c)=m and g^* (d)=n,

Then g^* is sb-continuous as well as contra sb-continuous

Also, g^* is b-continuous as well as locally closed continuous.

Theorem (2.4) : A mapping f : (X,T) → (Y, σ) is continuous iff f is b-continuous and locally closed continuous where (X,T) is extremely disconnected.

Proof:

Let f:(X,T) → (Y, σ) be a mapping from an extremely disconnected topological space(X,T)into another topological space (Y, σ).

Necessity: Let f be continuous & A ∈ σ . Then $f^{-1}(A)$ is open in (X,T). Using theorem (1.7), we have $f^{-1}(A)$ as a b-open and locally closed set. Consequently, f is a b-continuous & a locally closed continuous mapping

Sufficiency: Let f be b-continuous as well as locally closed continuous. Then, for an open subset A of Y, $f^{-1}(A)$ is b-open as well as locally closed set in X.

But (X,T) is extremely disconnected. So by Theorem (1.7) $f^{-1}(A)$ must be open. Equivalently, f is continuous.

Hence, the theorem

Theorem (2.5) : A mapping f : (X,T) → (Y, σ) is continuous iff f is b-continuous and D(c,b)-continuous.

Proof: Let f : (X,T) → (Y, σ) be a mapping from one topological space (X,T) to another (Y, σ).

Necessity:

Let f be continuous, then for an open set A of Y, $f^{-1}(A)$ is open in X. By theorem (1.8), $f^{-1}(A)$ must be b-open and a D(c,b) set. So, with the help of defns. of b-continuity and D(c,b)-continuity, it follows that f is also b-continuous as well as D(c,b)-continuous.

Sufficiency:

Let f be together b-continuous and D(c,b)-continuous. Then for an open set A of Y, $f^{-1}(A)$ is b-open and D(c,b)-set. This provides that $f^{-1}(A)$ is open and so f is continuous.

Hence, the theorem

Theorem (2.6) : A mapping $f:(X,T) \to (Y, \sigma)$ is contra b-continuous iff it is locally b-closed continuous and contra gb-continuous.

Proof: In order to establish the theorem, the following lemma is useful :

Lemma: If A is a subset of X where (X,T) is a topological space, then

[A is b-closed] $\Leftrightarrow$ [A is locally b-closed & gb-closed]

Proof of Lemma: Let A be a b-closed subset of X when (X,T) is a topological space, then $A = bcl(A)$

Also, $A = X \cap bcl(A)$, $X \in T$, Provides that A is locally b-closed.

Again, let U be an open set such that $A \subseteq U$.

Thus, we have, $bcl(A) \subseteq U$ whenever $A \subseteq U$ & U is open, which means that A is also gb-closed.

Conversely, let $A \subseteq X$ be locally b-closed & gb-closed. Clearly, there exists an open set U such that $A = U \cap bcl(A)$. This means that $A \subseteq bcl(A)$. But usually $bcl(A) \subseteq A$. Consequently, $A=bcl(A)$ i.e. A is a b-closed set.

Thus, [A is b-closed] $\Leftrightarrow$ [A is locally b-closed & gb-closed]

Proof of the main theorem:

Let G be an open set of Y whenever $f : (X,T) \to (Y, \sigma)$ is a mapping from a space (X,T) to other Space (Y, σ).

Now, from the above lemma,

[$f^{-1}(G)$ is b-closed] $\Leftrightarrow$ [$f^{-1}(G)$ is locally b-closed & gb-closed]

Therefore, $f : (X,T) \to (Y, \sigma)$ is contra b-continuous iff f is contra gb-continuous & locally b-closed continuous.

Hence, the theorem.

Theorem (2.7) : A mapping $f:(X,T) \to (Y, \sigma)$ is completely continuous iff it is b-pre-continuous and b-t-continuous

Proof: We, however, know that for a subset A of a space (X,T),

[A is regular open] $\Leftrightarrow$ [A is b-pre-open & b-t-set], [Theorem (1.12)]

Let G be an open subset of Y where $f : (X,T) \to (Y, \sigma)$ is a mapping. Then $f^{-1}(G) \subseteq X$. Now,

[$f^{-1}(G)$ is regular open] $\Leftrightarrow$ [$f^{-1}(G)$ is b-pre-open & b-t set] holds well.

Equivalently, f is completely continuous iff it is b-pre-continuous and b-t-continuous.

Hence, the theorem

Theorem (2.8) :

A mapping $f : (X,T) \to (Y, \sigma)$ is completely continuous iff it is b-pre-continuous and contra b-continuous.

Proof:

We, however, know that for a subset A of (X,T)

[A is regular open] $\Leftrightarrow$ [A is b-pre open & sb-generalized closed]. [Theorem (1.13)]

Let G be an open subset of Y where $f : (X,T) \to (Y, \sigma)$ is a mapping.

Then $f^{-1}(G) \subseteq X$

Now,

[$f^{-1}(G)$ is regular open] $\Leftrightarrow$ [$f^{-1}(G)$ is b-pre-open & sb-generalized closed] holds good.

Equivalently, f is completely continuous iff it is b-pre-continuous and contra sb-continuous.

Hence, the theorem

3. STRONGLY b-CONTINUOUS FUNCTIONS:

The notion of strongly b-continuous mapping is introduced and developed to study their characteristic property and some interesting results in this section.

Definition (3.1) :

A mapping $f : (X,T) \to (Y, \sigma)$ from a topological space (X,T) to another topological space (Y, σ) is said to be strongly b-continuous (briefly sg-b-continuous) on X iff $f^{-1}(\sigma\text{-int } A) = \text{b-int}(f^{-1}(A))$; $\forall A \subseteq Y$.

Example (3.2) :

Let $f : (X,T) \to (Y, \sigma)$ be a mapping from one topological space (X,T) into another topological space (Y, σ), where $X = \{a,b,c\}$, $T = \{\phi, x, \{a\}, \{a,b\}\}$ & $Y = \{l,m\}$, $\sigma = \{\phi, \{l\}, Y\}$ such that

$f(a) = l$, $f(b) = m = f(c)$.

Here, $Bo(X,T) = \{\phi, x, \{a\}, \{a,b\}, \{a,c\}\}$

Now,

$f^{-1}(\sigma\text{-int}\{l\}) = f^{-1}(l) = \{a\} = \text{bint }\{a\} = \text{bint}(f^{-1}\{l\})$

$f^{-1}(\sigma\text{-int}\{m\}) = f^{-1}(\phi) = \phi$; $\text{bint } f^{-1}\{m\} = \text{bint}\{b,c\} = \phi$

$f^{-1}(\sigma\text{-int}\{l,m\}) = f^{-1}(Y) = X$; $\text{bint}(f^{-1}\{l,m\}) = \text{bint}(X) = X$

So, $f^{-1}(\sigma\text{-int}A) = \text{bint}(f^{-1}(A))$ for every subset A of Y which means that f is strongly b-continuous on X.

However, if we consider the topology $T^* = \{\phi, X, \{a\}, \{b\}, \{a,b\}, \{b,c\}\}$ on X and other definitions as above, we see that $Bo(X,T^*) = T^*$ and $\text{b-int}(f^{-1}(\{m\})) = \text{bint}\{b,c\} = \{b,c\}$ but $f^{-1}(\sigma\text{-int}\{m\}) = f^{-1}(\phi) = \phi$.

i.e. $f^{-1}(\sigma\text{-int}\{m\}) \neq \text{bint}(f^{-1}(\{m\}))$.

This proves that $f : (X,T^*) \to (Y, \sigma)$ is not a strongly σ-continuous mapping.

It is important to note that f is b-continuous in both the cases.

Remark (3.3):

The above fact enables us to draw the conclusion as :

If $f : (X,T) \to (Y, \sigma)$ be a mapping defined on (X,T) into (Y, σ) be a mapping defined on (X,T) into (Y, σ), then

f is sg-b-continuous $\Rightarrow$ f is b-continuous

$\Rightarrow$ f is continuous

Now, the following theorem appears as the charcteristic property of strongly b-continuity.

Theorem (3.4) :

A mapping $f : (X,T) \to (Y, \sigma)$ of a topological space (X,T) into another topological space (Y, σ) is strongly b-continuous on X if and only if $f^{-1}(\sigma\text{-cl}(A)) = bcl(f^{-1}(A))$, for every subset A of Y.

Proof:

Suppose that f is a strongly b-continuous mapping on X, then $f^{-1}(\sigma\text{-int}(A)) = bint(f^{-1}(A))$; V $A \subseteq X$ is for $A \subseteq Y$,

We have,

$$\begin{aligned} f^{-1}(\sigma\text{-cl}(A)) &= f^{-1}(\{\sigma\text{-int}(A^c)\}^c) \\ &= \{f^{-1}(\sigma\text{-int}(A^c)\}^c \\ &= \{bint(f^{-1}(A^c))\}^c \\ &= \{bint(f^{-1}(A))^c\}^c \\ &= bcl(\{(f^{-1}(A))^c\}^c) \\ &= bcl(f^{-1}(A)) \end{aligned}$$

Conversely, suppose that $f^{-1}(\sigma\text{-cl}(A))= bcl(f^{-1}(A))$ for subset A of Y.

Now, $$\begin{aligned} f^{-1}(\sigma\text{-int}(A)) &= f^{-1}(\{\sigma\text{-cl}(A^c)\}^c) \\ &= \{f^{-1}(\sigma\text{-cl}(A^c)\}^c \\ &= \{bcl(f^{-1}(A^c))\}^c \\ &= bint\ \{(f^{-1}(A^c))^c\} \\ &= bint\ \{(f^{-1}(A^c)^c)\} \\ &= bint(f^{-1}(A)). \end{aligned}$$

Hence, f is strongly b-continuous on X.

Corollary (3.5) : If $f : (X,T) \to (Y, \sigma)$ is sg-b-continuous on X, then

(i) $f^{-1}(\sigma\text{-int}(\sigma\text{-cl}(A))) = bint(bcl(f^{-1}(A)))$

(ii) $f^{-1}(\sigma\text{-cl}(\sigma\text{-int}(A))) = bcl(bint(f^{-1}(A)))$, V $A \subseteq Y$.

Corollary (3.6): If $f : (X,T) \to (Y, \sigma)$ is strongly b-continuous on X, then the following results hold good :

a) If A is p-open in Y, then $A \subseteq \sigma\text{-int}(\sigma\text{-cl}(A))$

$\Rightarrow$ $f^{-1}(A) \subseteq f^{-1}(\sigma\text{-int}(\sigma\text{-cl}(A)))$

$\Rightarrow$ $f^{-1}(A) \subseteq bint(bcl(f^{-1}(A)))$ .... (i)

Now the following cases arise :

a_1) When (X,T) is extremelly disconnected i.e. T-cl(U) is open for every open subset U of X, we have Bo(X,T) = PO(X,T) and hence from (i)

$f^{-1}(A) \subseteq bint(bcl(f^{-1}(A))) = pint(pcl(f^{-1}(A)))$

$\Rightarrow$ $f^{-1}(A)$ is also p-open in X.

a_2) When (X,T) is strongly irresolvable i.e. every open subspace of (X,T) is irresolvable i.e. every open subspace of (X,T) has no two disjoint dense subsets, we have Bo(X,T)=SO(X,T) and so from(i)

$f^{-1}(A) \subseteq bint(bcl(f^{-1}(A))) = sint(scl(f^{-1}(A)))$

$\Rightarrow$ $f^{-1}(A)$ is also β-open

b) If A is s-open in Y, then $A \subseteq \sigma\text{-cl}(\sigma\text{-int}(A))$

$\Rightarrow$ $f^{-1}(A) \subseteq f^{-1}(\sigma\text{-cl}(\sigma\text{-int}(A)))$

$\Rightarrow$ $f^{-1}(A) \subseteq bcl(bint(f^{-1}(A)))$ .... (ii)

Here, the following cases arise:

b_1) When (X,T) is extremely disconnected, we have

Bo(X,T) = PO(X,T) and hence from (ii)

$$f^{-1}(A) \subseteq bcl(bint(f^{-1}(A))) = pcl(pint(f^{-1}(A)))$$

$$\Rightarrow f^{-1}(A) \text{ is b-open.}$$

b_2) When (X,T) is strongly irresolvable we have

BO(X,T) = SO(X,T) and so from (ii)

$$f^{-1}(A) \subseteq bcl(bint(f^{-1}(A))) = scl(sint(f^{-1}(A)))$$

$$\Rightarrow f^{-1}(A) \text{ is s-open.}$$

CONCLUSION:

The basic important properties of decomposition of continuity via locally b-closed, b-pre-open, sb-generalized closed sets have been obtained and their interrelationship has been analyzed toopen a new horizon in the world of Mathematics. The future scope of study is to obtain results regarding invariance & topological property of respective compactness under such mappings. Strongly b-continuous function has been discussed in the context of extremely disconnected & strongly irresolvable spaces.

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